

# $\alpha^*$ -Approximated Delaunay Triangulation Based Descriptors for Handwritten Character Recognition

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**Abstract**—This paper presents an original feature vector extraction process based on the Delaunay triangulation (DT) and a zoning technique. The presented work provides an illustration of the equivalency between a zoning and the Delaunay triangulation in the context of handwritten character recognition. A novel technique that relies on the approximation of a DT and an automatic pruning calculation is introduced. We call this technique the  $\alpha$ -approximation. To discuss our contribution, experiments are conducted on the MNIST database of handwritten digits using a support vector machine classifier for the classification task.

## I. INTRODUCTION

Optical character recognition (OCR) is the process that converts scanned documents, either printed or handwritten, into electronic versions. Typical uses are content-based document search or document classification. OCR field has been active for several decades, leading at the present time to efficient recognition systems, especially for isolated characters. However, due to the large variability of scriptwriters, and therefore of handwritten patterns, the recognition of handwritten characters still remains an open problem [1]. It is a difficult task as it relies on the relevance of the combination: image type, feature vector and classifier.

In the domain of machine learning, objects classifying is a very typical task. A canonical scenario consists of training a model on a given data set, for which we know the actual label of each element. After this learning phase, classification of unknown objects can be proceeded. In the context of OCR, the description of the characters in a manner suitable for classification is a crucial issue as it depends on the quality of the low-level information extracted. Moreover, the specificity of the used classifier is an arguable point. Regarding the former problem, which we address in this paper, the common scheme for isolated character recognition is composed of: (i) the description of the characters by a set of features (typically) embedded into a vector, (ii) the classification of the characters, described in the space of the feature vectors. Many features have been proposed and used for character recognition [2]. Popular ones can be categorized in the following three types: (i) *Statistical*

*features* that describe the distribution of points (e.g. zoning, projections, profiling techniques), (ii) *Structural features* that describe geometrical properties of the character (e.g. holes, loops, branch points, strokes), (iii) *Space transformation based features* (e.g. Fourier transform, moments). The methods we propose and discuss in this paper extract zoning-based statistical features, computed out of the pixels of the character on one hand, and out of the Delaunay triangulation (DT) of these pixels on the other.

One of the objectives of this paper is to provide an illustration of the equivalency between the compressed quadtree structure and the DT of a planar point set [3], in the context of handwritten character recognition. For this purpose, we present and compare two methods of feature extraction. The first one is based on a classical uniform static zoning technique, which we can assimilate to the compressed quadtree structure [4], and the second one is based on the construction of the DT.

Another objective is to provide a method extracting features which tend to carry *local* and *less local* information of an handwritten character. To achieve this objective, the introduced zoning technique concatenates the statistical information retrieved from uniform zonings that were operated on several levels of resolution. We call this concatenation process *multilevel zoning*. Besides, the introduced DT-based feature extraction method exploits the information carried by the simplices of the DT. Indeed, the DT allows to embed both density and shape information of the character. As well, in dimension 2, the number of simplices equals at most  $6n$ , with  $n$  the number of input points [5]. Thus, the construction of the DT allows to upsample the input data point set, enriching then the amount of information to be considered in the feature vector extraction process.

The rest of the paper is constructed as follows. In the next section, we briefly recall some basic concepts about the DT, and define the notion of  $\alpha$ -approximation of a DT. In Section III, we formally define the zoning technique, the neighborhood strategy, the proposed *multilevel zoning* method and the  $\alpha$ -approximated DT-based method. In Section IV, we present the experimental results we achieved and discuss the

efficiency of the proposed method. We conclude in Section V.

## II. $\alpha$ -APPROXIMATION OF THE DELAUNAY TRIANGULATION

### A. Delaunay triangulation

The Delaunay triangulation (DT) [5] is a widely-used structure that allows to grasp geometrical and topological relationship between given  $d$ -dimensional points. Let  $z_1, \dots, z_n$  be points in  $S \subseteq \mathbb{R}^d$  and let  $V_i$  be the Voronoi cell of  $z_i$ , composed of all the points in  $\mathbb{R}^d$  for which  $z_i$  is the closest element:  $V_i = \{x \in \mathbb{R}^d \mid \|x - z_i\| \leq \|x - z_j\|, \forall j \neq i\}$ .  $V_i$  is the intersection of  $n - 1$  hyperplanes and the *Voronoi Diagram* of  $S$  is the set of the Voronoi cells  $\text{Vor}S = \{V_i \mid 1 \leq i \leq n\}$ . The Delaunay triangulation of point set  $S$ ,  $DT(S)$ , dual to the Voronoi diagram, is defined by connecting every two points for which their associated Voronoi cell share a common face. We consider here the points of  $S$  in a non-degenerate configuration, i.e. there is no  $d + 2$  points lying on any circumscribed hyper-sphere. From now on, we call *triangles* the 2-simplices.

### B. $\alpha$ -approximation with triangles exclusion

Let  $S$  be a pixel point set associated to a given character. We present here the  $\alpha$ -approximation of  $DT(S)$  and the proposed triangle-pruning strategy. Before defining what an approximation is, we first note that there are two types of triangles, depending on whether they are inside or bridging (inner or outer) gaps between parts of the character. The triangles inside the character present significantly lower perimeter values and the ones bridging gaps present more *heterogeneous* edges length distribution as along with a larger perimeter. On the criterion of the perimeter value or the so-called *heterogeneity*, it is thus possible to construct an ordering that discriminates the two types of triangles. This measure-dependent ordering provides a basis on which we define a triangle-pruning strategy: the higher the value of the measure, the more the associated triangle is prone to be pruned. This ordering implicitly defines a basis for the approximation of the actual  $DT(S)$ .

Let  $\Sigma > 0$  be the number of simplices of  $DT(S)$  and  $\mu$  a measure defined on the triangles of  $DT(S)$ . Given measure  $\mu$  and triangles  $(s_i)_{1 \leq i \leq \Sigma}$  of  $DT(S)$ , we define  $O_\mu^{DT(S)}$  the ordered set of scalar values

$$O_\mu^{DT(S)} = \{\alpha_1, \dots, \alpha_\Sigma\}$$

for which  $0 < \alpha_i \leq \alpha_{i+1} < +\infty$  and  $\alpha_i = \mu(s_i), \forall i \in [1, \Sigma]$ . Given the set  $O_\mu^{DT(S)}$ , we then define the set of triangulations  $I = \{DT(S)_{\alpha_1}, \dots, DT(S)_{\alpha_\Sigma}\}$  where each  $DT(S)_\alpha$  is a Delaunay triangulation associated to the pruning value  $\alpha$ . Each triangle  $s$  of  $DT(S)_\alpha$  is such that  $\mu(s) \leq \alpha$ . As a consequence, we note that  $\alpha_i \leq \alpha_{i+1} \Rightarrow DT(S)_{\alpha_i} \subseteq DT(S)_{\alpha_{i+1}}$ . It follows that the triangulations of  $I$  are such that  $DT(S)_0 \subseteq DT(S)_{\alpha_1} \subseteq \dots \subseteq DT(S)_{\alpha_\Sigma}$ .

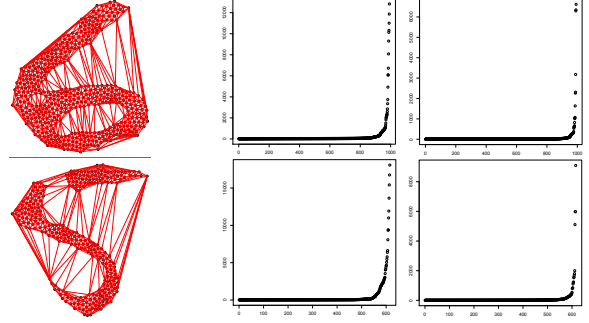


Figure 1. **Delaunay triangulation and the measures of the triangles.** Delaunay triangulation computed on two characters (left) from the MNIST handwritten digits database and their respective ordered set  $O_\mu^{DT(S)}$  with  $\mu = \mu_{perim}$  (center) and  $\mu = \mu_{heter}$  (right).

$DT(S)_0$  equals the original point set  $S$  and  $DT(S)_{\alpha_\Sigma}$  the original triangulation  $DT(S)$ . A given value  $\alpha \in O_\mu^{DT(S)}$  thus defines the  $\alpha$ -approximation of  $DT(S)$ , with respect to measure  $\mu$ , as  $DT(S)_\alpha$  contains no triangle for which the measure is greater than  $\alpha$ . The efficiency of the  $\alpha$ -approximation of  $DT(S)$  relies on the correct association of a measure to the structure of the triangulation. As the approximation procedure goes along with pruning, this measure may exhibit geometrical information [6]. In order to investigate which triangles of  $DT(S)$  do carry relevant information of the character, we studied the impact of two pruning measures on the classification process results. Given a triangle  $s$  and its edge set  $E_s$ , we studied the perimeter and *heterogeneity*:

$$\mu_{perim}(s) = \sum_{e \in E_s} |e|, \quad \mu_{heter}(s) = \mu_{perim}(s) * \frac{\max E_s}{\min E_s}.$$

Derived from discussion in [6], measure  $\mu_{heter}$  gives higher values to triangles which are longer and which bridge inner or outer gaps of the character. These triangles tend to have higher values for both  $\mu_{perim}$  and  $\mu_{heter}$ . The latter measure allows to differentiate triangles which present equal perimeter values by pruning first the ones which bridge gaps of the character as the bridging-gap triangles tend to present more heterogeneous edge length distribution. This remark is illustrated in Figure 1.

### C. Computation of $\alpha^*$ , a heuristic

Given the shapes of the curves presented in Figure 1, finding inner triangles boils down to finding all the ones presenting a lower perimeter and *heterogeneity* value. In order to study the influence of the triangles outside of the character on the classification task, we propose the following strategy. We define a curvature criterion on the ordered set  $O_\mu^{DT(S)}$ ,

$$\gamma(x) = \frac{y''}{(1 + y'^2)^{3/2}}$$

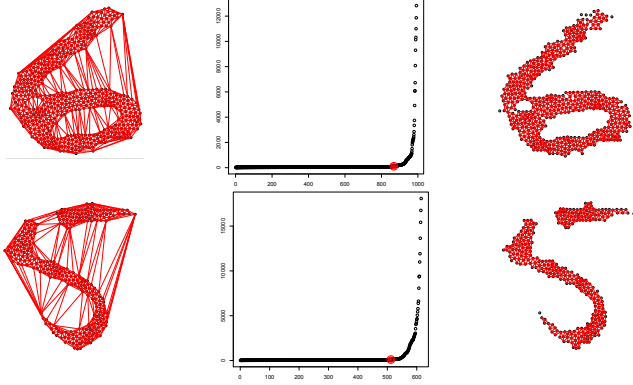


Figure 2.  $\alpha$ -approximation and optimal pruning. (From left to right)  $DT(S)$ , selected  $i^*$  (red dot) with the perimeter pruning measure and the  $\alpha^*$ -approximation of  $DT(S)$ .

where  $x \in \llbracket 1, \Sigma \rrbracket$  and  $y \in O_\mu^{DT(S)}$ ,  $y'$ ,  $y''$  define the measured values, the first and second derivative of  $y$  respectively.

We set as a pruning strategy the exclusion of every triangles whose index is greater than  $i^*$ , the index of optimal  $\alpha^*$ . A naive strategy may consist in finding the index of the maximum curvature value in  $O_\mu^{DT(S)}$  but it may result in favoring indices associated to high values. To bypass this problem, we define the set of windows  $W = \cup_{i \in \llbracket 1, \Sigma - m \rrbracket} W_i$ , where  $W_i = \{w_1^i, \dots, w_m^i\}$ .  $w_k^i \in \llbracket 1, \Sigma \rrbracket$  are the indices of window  $W_i$  and  $m$  is the size of the windows. It follows that  $i^* = \operatorname{argmax}_{i \in \{1 \dots |W|\}} \gamma_i$ , where  $|W| = \Sigma - m + 1$  is the number of windows defined on  $O_\mu^{DT(S)}$  and

$$\gamma_i = \gamma(W_i) = \frac{\frac{1}{m} \sum_{w \in W_i} \gamma(w)}{\max_{w \in W_i} \gamma(w)}.$$

The introduced curvature criterion  $\gamma_i$  allows then to select the maximum curvature of the curve according to input parameter  $m$ . An illustration of  $\alpha$ -approximation following the presented strategy is given in Figure 2, along with the position of the associated  $\alpha^*$  in the curve.

### III. ZONING BASED FEATURE VECTOR EXTRACTION PROCESS

#### A. The zoning technique

Zoning is a well-known region-based feature extraction technique in pattern recognition field. Several zoning methods have been proposed, using uniform and non-uniform grids. More recently, zoning design has been considered as an optimization problem, resulting in various dynamic zoning design techniques [7]. Zoning technique applied to handwritten character recognition has given very relevant results [7], leveraging the region-based feature extraction approaches ability to handle a large variability of handwritten patterns.

Let  $I$  be an image. A static uniform zoning method define an  $M \times N$  axis-aligned regular grid layered on  $I$ . A zoning method can be considered as a partition of  $I$  into  $M \times N$

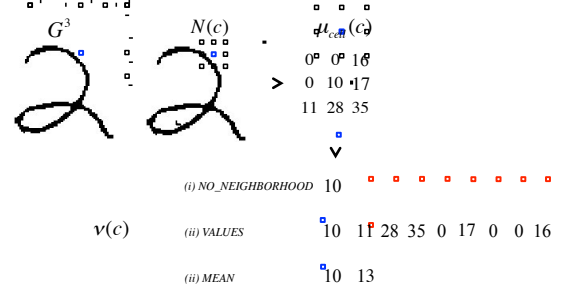


Figure 3. **Neighborhood strategies.** Possible vectors  $\nu(c)$  for a given cell  $c \in G^3$ , regarding the 3 proposed neighborhood strategies.

zones or cells. We define a grid  $G^k = \{c_{ij}^k, \forall i, j \in [1..2^k]\}$ , with  $k$  the order of the zoning. Each cell  $c^k \in G^k$  provides statistical information of a part of  $I$ .

Classical zoning methods calculate the number of elements inside each cell of the grid. This information can be enhanced by taking into account the density of the points in a neighborhood of a given cell  $c^k$ . We define a 8-neighborhood centered around  $c^k$ :  $N(c^k) = \{c_v^k, \forall v \in [1..V]\}$ , with  $V \in \{3, 5, 8\}$ , depending on the position of  $c^k$  in  $G^k$ , i.e. in a corner, along the border or inside respectively. We note  $\nu(c^k)$  the vector containing the retrieved information for the cell  $c^k$ ,  $\mu_{cell}$  the measure extracted within the cell  $c^k$  and  $|c^k|$  the number of elements inside  $c^k$ .

#### B. The neighborhood strategies

Three neighborhood strategies have then been studied to fill the vector  $\nu(c^k)$ : the *NO\_NEIGHBORHOOD* strategy, which put  $|c^k|$  in  $\nu(c^k)$ , the *VALUES* strategy, which appends  $|c^k|$  to  $\{|c|, \forall c \in N(c^k)\}$  in  $\nu(c^k)$  and the *MEAN* strategy, which appends  $|c^k|$  to the mean value of  $\mu_{cell}$  over  $c^k \cup N(c^k)$  in  $\nu(c^k)$ . The size of vector  $\nu(c^k)$  regarding the first, second and third strategy is 1,  $|N(c^k)| + 1$  and 2 respectively. Figure 3 illustrates the feature extraction for one cell.

Finally, the vectors  $\nu(c^k)$  are appended to build the feature vector  $FV^k$ , which is the actual descriptor of  $G^k$ . The size of vector  $FV^k$  is  $2^{2*k}$ ,  $\sum_{c^k \in G^k} (|N(c^k)| + 1)$  and  $2^{2*k+1}$  for strategies *NO\_NEIGHBORHOOD*, *VALUES* and *MEAN* respectively. The defined *MEAN* strategy provides an acceptable trade-off between the size of  $FV^k$  and the amount of embedded neighborhood information.

#### C. The multilevel zoning

For a given order  $k$ , the defined feature vector  $FV^k$  constitutes a global descriptor of the image  $I$ . We propose to enhance the description of  $I$  by embedding information at several levels of resolution. A *multilevel* approach is introduced. For a given order  $K > 1$ ,  $K$  vectors  $FV^k$  are computed for every  $k$  in  $[1..K]$ . These vectors are then appended to build the actual feature vector  $FV$  of  $I$ . Therefore, the size  $|FV|$  of vector  $FV$  is  $\sum_{k \in [1..K]} |FV^k|$ .

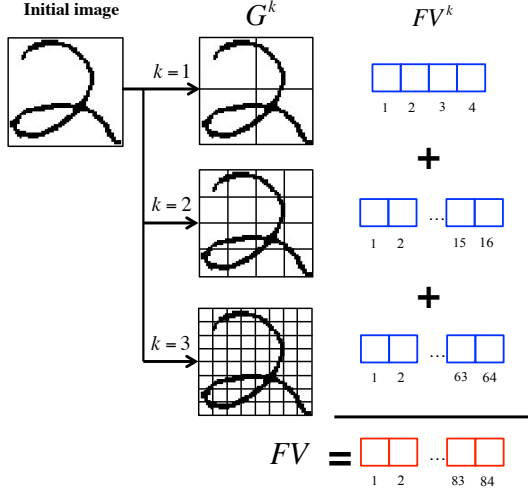


Figure 4. **The multilevel zoning technique based feature vector extraction.** Construction of the final feature vector  $FV$  for  $K = 3$ .

For  $K = 3$  and the *MEAN* strategy,  $|FV| = 2 \times (2^3 \times 2^3) + 2 \times (2^2 \times 2^2) + 2 \times (2^1 \times 2^1) = 168$ . Figure 4 illustrates the multilevel zoning feature vector construction. It is important to note that using the multilevel approach  $|FV|$  grows exponential with respect to  $K$ . In this paper, we classically set the number of elements in a cell  $c^k \in G^k$  as a measure, i.e  $\mu_{cell}(c^k) = |c^k|$ .

#### D. The $\alpha$ -approximated DT and input data selection

We finally present the construction of a feature vector based on the  $\alpha$ -approximated DT. As illustrated in Figure 2, the  $\alpha$ -approximation of the DT tends to exhibit triangles describing the inner part of a character, regarding the measure  $\mu_{perim}$  and a pruning value  $\alpha^*$ . Measure  $\mu_{heter}$  tends to as well. Once the  $\alpha$ -approximation is built, the previously defined zoning technique is used to construct the actual DT-based feature vector  $FV^k$ . We consider two types of input elements for a given cell  $c^k$ . First, the centers of gravity of each triangles. Secondly, both raw data (i.e. input points that describe a character) and centers of gravity are taken into consideration. This allows to upsample the input data point set, enriching then the amount of information to be considered in the feature vector extraction process.

## IV. EXPERIMENTS.

### A. Dataset and pre-processing

Experiments were carried out on the 10 digit classes of the MNIST isolated handwritten database<sup>1</sup>. Each image has been cropped to the bounding box of the digit, deleting the white border, and has been surrounded with a 1-pixel margin. To allow the study of more orders of zoning, each sample has then been magnified to have a final size of

<sup>1</sup><http://yann.lecun.com/exdb/mnist/>

Table I  
COMPARISON OF THE ACHIEVED RECOGNITION RATES FOR ORDER  $K = 4$  WITH RESPECT TO THE NEIGHBORHOOD STRATEGY.

Strategy	<i>NO_NEIGHBORHOOD</i>	<i>VALUES</i>	<i>MEAN</i>
<b>Recognition Rate</b> ( $ FV^4 $ )	93,99 (256)	95.21 (2116)	96.41 (512)

$128 \times 128$ . Finally, a skew and slant normalization has been done using the method proposed in [8]. The average number of black pixels over the 5000 images of the 10 classes is 4362. Throughout the presented experiments, we reduced the size of the input data points to 10% by selecting a *data-representative* subset using *k-means* algorithm. A Gaussian Kernel with Radial Basis Function SVM classifier<sup>2</sup> has been trained and recognition rates have been obtained using a 5-fold cross-validation technique.

### B. Feature Extraction: multilevel zoning technique vs. DT

The introduced neighborhood strategies are first discussed. We set here the order value  $K$  to 4, as (i) it allows to build a feature vector of a reasonable size and (ii) Table II reports that the recognition rates decrease from  $K = 5$ . As exposed in Table I, the best compromise between the size of the feature vector and the recognition rate is provided by strategy *MEAN*. Let us compare now the zoning based descriptor to the DT-based one. The objective here is to provide an illustration of the equivalency between the classical feature extraction zoning technique processed on the raw data and the introduced DT-based feature extraction. This objective is inspired by results presented in [3]. As we can observe in Table II, the recognition rates achieved from the DT-based zoning technique (*DT NO\_ML*) are equivalent to the ones from a classical zoning technique (*NO\_DT NO\_ML*), as an expected result derived from [3]. Indeed, we can observe that the recognition rates growth behave the same. These remarks hold in the multilevel case (*ML*).

### C. Exploiting the $\alpha$ -approximation of the DT

Let us now investigate the information carried by the DT itself. The objective here is to study the impact of the shape

Table II  
RECOGNITION RATES ACHIEVED FOR THE CLASSICAL ZONING APPROACH AND THE DT-BASED ONE WITH THE STRATEGY *MEAN*.

ORDER	1	2	3	4	5
<b>DT NO_ML</b> (vect. size)	56.77 (8)	91.73 (32)	95.60 (128)	95.94 (512)	95.40 (2048)
Growth rate (%)	-	+61.59	+4.22	+0.36	-0.56
<b>NO_DT NO_ML</b> (vect. size)	57.38 (8)	92.09 (32)	96.17 (128)	96.41 (512)	96.21 (2048)
Growth rate (%)	-	+60.49	+4.43	+0.25	-0.2
<b>DT ML</b> (vect. size)	56.77 (8)	91.28 (40)	95.45 (168)	96.12 (680)	96.37 (2728)
Growth rate (%)	-	+60.79	+4.57	+0.70	+0.26
<b>NO_DT ML</b> (vect. size)	57.38 (8)	91.81 (40)	96.05 (168)	97.19 (680)	97.13 (2728)
Growth rate (%)	-	+60	+4.62	+1.12	-0.06

<sup>2</sup><http://www.csie.ntu.edu.tw/%7Eejlin/libsvm/>

Table III

INFLUENCE OF THE INPUT POINTS, THE PRUNING MEASURE AND THE PRUNING VALUE ON THE RECOGNITION RATE WITH THE STRATEGY *MEAN*. THE RATES WERE COMPUTED FOR ORDER  $K = 4$ .

$\mu$	<i>perimeter</i>				<i>heterogeneity</i>			
$\alpha$	0	0.1	0.5	$\alpha^*$	0	0.1	0.5	$\alpha^*$
<b>CG</b>	96.2	96.1	89.1	94.4	96.2	96	94.8	96.3
<b>CG_RD</b>	96.6	96.5	94.9	96.2	96.6	96.3	96.2	96.6

and position of the triangles on the recognition rate. As well, we conduct a comparison between taking into account or not the raw data points as an input of the feature vector extraction process.

Table III reports the sensitivity of the recognition rate regarding the following three points: (i) setting the centers of gravity of the triangles along with the raw data (*CG\_RD*) as an input of the feature vector extraction process or the centers of gravity alone (*CG*), (ii) setting  $\mu_{perim}$  or  $\mu_{heter}$  (defined in Section II.B.) as a pruning measure, (iii) the value  $\alpha$  of the pruning measure. For  $\alpha^*$  calculation, we set the size of windows  $W_i$  to  $\frac{\Sigma}{10}$  with  $\Sigma$  the number of triangles of  $DT(S)$ .

We can observe that, regardless the pruning measure and the pruning value, the results obtained by considering the input points along with the computed centers of gravity gives the best results. This is an expected consequence of enriching the geometrical information carried by the triangles along with the one of the input points. Note that the recognition rate values are the same for the two measures when  $\alpha = 0$  as no pruning is done. The results reported in Table III also highlight the fact that the *heterogeneity* pruning measure gives better results than the *perimeter* one does. This confirms the fact that the ordering based on the *heterogeneity* measure allows to keep more relevant triangles from the DT: large and heterogeneous-edges-presenting triangles are pruned first. Furthermore, Table III shows that, whatever the measure, the recognition rate decreases as the value of  $\alpha$  grows, as an expected consequence of pruning more and more triangles. For the *heterogeneity* measure, the best score is achieved using the proposed  $\alpha^*$ -approximation of the DT. Table IV exposes the precision and recall values per class. Class 7 presents the lowest value of precision and is the most predicted class. Class 9 presents the lowest recall value and is the most misclassified one.

Note that for the *perimeter* measure,  $\alpha^*$  underperforms the other values of  $\alpha$ . This can be explained by the fact that the implied ordering is prone to put for deletion triangles which are part of the inner part of the characters. Besides, we suspect that a better tuning of the  $\alpha^*$  calculation for the *perimeter* measure might sort this problem out, as the proposed heuristic experimentally pruned more triangles for this measure.

## V. CONCLUSION

The presented work provides an illustration of the equivalency between the quadtree structures and the Delaunay

Table IV

COMPARISON OF THE ACHIEVED PRECISION AND RECALL VALUES FOR EACH DIGIT COMPUTED FOR: THE STRATEGY *MEAN*, *CG\_RD* INPUT POINTS, *HETEROGENEITY* PRUNING MEASURE AND PRUNING VALUE  $\alpha^*$ .

<b>DIGIT</b>	0	1	2	3	4
<b>Precision (%)</b>	99.2	96.4	96.8	97.3	94.8
<b>Recall (%)</b>	97.8	97.4	96.8	96.4	96
<b>DIGIT</b>	5	6	7	8	9
<b>Precision (%)</b>	96.6	98	94.5	97.5	94.7
<b>Recall (%)</b>	97.2	98.8	96	95.1	94.3

triangulation in the context of handwritten character recognition. Indeed, the non multilevel zoning and the DT-based zoning techniques provided equivalent results as they both encapsulate equivalent information. We proposed a feature extraction process based on a novel and promising technique: the  $\alpha$ -approximation of a DT. This technique relies on the association of a measure and a pruning technique. An automatic pruning value strategy has been provided as well. Experiments showed that leveraging information carried by the DT slightly improves the recognition rate. Future works will focus on studying new pruning measure  $\mu$  and extracting information regarding topological configuration of the gaps outside the character.

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